B.SC. FOURTH SEMESTER (HONOURS) EXAMINATIONS, 2021

Subject: Mathematics **Course ID: 42115**

Course Code: SH/MTH /405/SEC-2 **Course Title: Graph Theory**

Full Marks: 40 **Time: 2 Hours**

<u>The figures in the margin indicate full marks</u>

Notations and symbols have their usual meaning.

1. **Answer *any five* of the following questions:** **(2×5=10)**

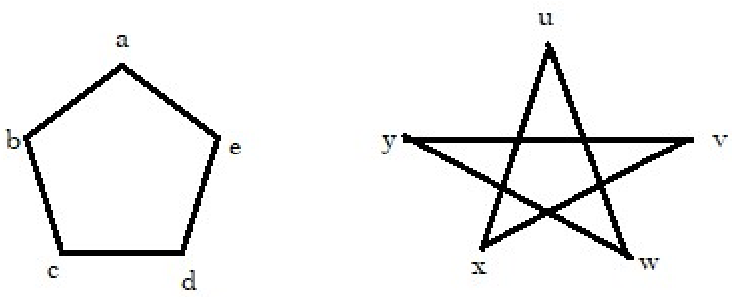
a) Justify whether it is possible or not to draw a graph with 12 vertices having (i) 9 edges, (ii) 13 edges.

b) Does there exist a simple graph with six vertices having degrees 5,5,4,2,2,2? Justify your answer.

c) How many vertices are there in a graph with 20 edges if each vertex is of degree 5?

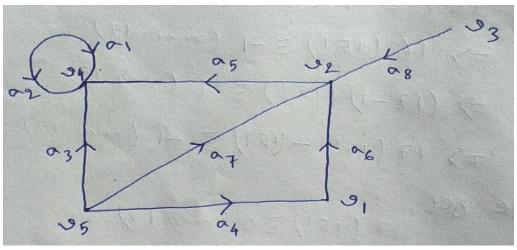
d) Give an example of a Hamiltonian graph which is not Eulerian.

e) Define isomorphism. Determine whether the following pair of graphs are isomorphic.



f) Find the minimum and maximum number of edges of a connected graph with 10 vertices.

g) Find the adjacency matrix of the following di-graph.



h) Why do the adjacency matrix of a simple graph G is orthogonally diagonalizable? Explain.

2. **Answer *any four* of the following questions:** **(5×4=20)**

a) Prove that a graph G is connected if and only if it has a spanning tree.

b) Show that the number of spanning trees of K_n is n^{n-2}.

c) (i) Define radius $r(G)$ and diameter $d(G)$ of a graph G.

(ii) Prove that $r(G) \leq d(G) \leq 2r(G)$. 2+3

d) Let G be a connected graph. Then prove that G will be Eulerian if and only if it can be decomposed into circuits.

e) (i) Prove that a graph G has a spanning tree if and only if G is connected. 3+2

(ii) Show that a complete graph K_n is Eulerian if n is odd.

f) Let T be a tree with n vertices, $n \geq 1$. Then prove that T has exactly $(n-1)$ edges.

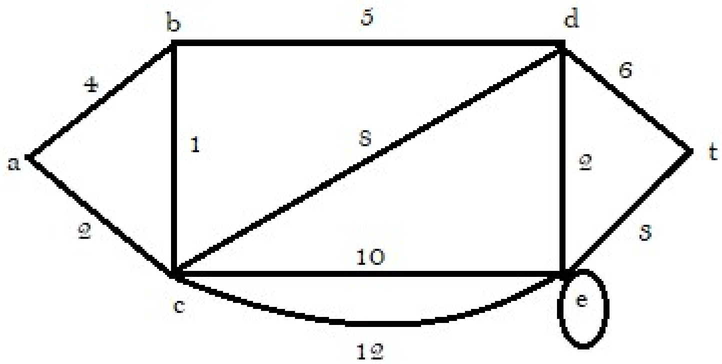
3. **Answer *any one* of the following questions:** **(10×1=10)**

a) (i) Prove that a simple graph G is Hamiltonian if and only if its closure $C(G)$ is Hamiltonian.

(ii) A medical representative has to visit five stations. He does not like to visit any station twice before completing his tour of all stations. The costs of going from one station to another are given below. Determine the optimal route and the minimum expenditure. (5+5)

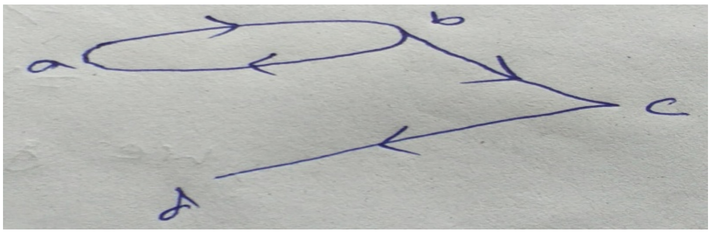
	5	8	4	5
5		7	4	5
8	7		8	6
4	4	8		8
5	5	6	8	

b) (i) Using Dijkstra's Algorithm find the length of the shortest path of the following graph from the vertex a to t :



(ii) Let $G = (V, E)$ be a connected graph. Then prove that the distance function $d(u, v)$ is a metric defined on V for $u, v \in V$.

(iii) Find the path matrix for the di-graph given below



(5+3+2)

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